

Precalculus, Section 1.2, #51
Intercepts and Symmetry

For the following equation¹, list the intercepts and test for symmetry.

$$9x^2 + 4y^2 = 36$$

To find the y -intercept(s), we substitute $x = 0$ and solve for y .

$$9(0)^2 + 4y^2 = 36$$

$$0 + 4y^2 = 36$$

$$4y^2 = 36$$

$$\frac{1}{4} \cdot 4y^2 = \frac{1}{4} \cdot 36$$

$$y^2 = 9$$

$$\sqrt{y^2} = \sqrt{9}$$

$$\pm y = 3$$

$$y = 3 \text{ or } -y = 3$$

$$y = 3 \text{ or } y = -3$$

Since $\sqrt{a^2} = |a|$ and
 $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$,
 $\sqrt{a^2} = a$ or $-a$.
We often write this as $\sqrt{a^2} = \pm a$.

Thus, the y -intercepts are $(0,3)$ and $(0, -3)$.

Similarly, to find the x -intercept(s), we substitute $y = 0$ and solve for x .

$$9x^2 + 4(0)^2 = 36$$

$$9x^2 + 0 = 36$$

$$9x^2 = 36$$

$$\frac{1}{9} \cdot 9x^2 = \frac{1}{9} \cdot 36$$

$$x^2 = 4$$

$$\sqrt{x^2} = \sqrt{4}$$

$$\pm x = 2$$

$$x = 2 \text{ or } -x = 2$$

$$x = 2 \text{ or } x = -2$$

Thus, the x -intercepts are $(2,0)$ and $(-2,0)$.

The graph of $9x^2 + 4y^2 = 36$ is symmetrical about the y -axis if both x and $-x$ give the same y -value. We substitute $-x$ into the equation for x and simplify.

$$9(-x)^2 + 4y^2 = 36$$

and since $(-x)^2 = (-1 \cdot x)^2 = (-1)^2 \cdot x^2 = 1 \cdot x^2 = x^2$ we get

$$9x^2 + 4y^2 = 36$$

Thus the graph of $9x^2 + 4y^2 = 36$ is symmetric with respect to the y -axis.

¹Sullivan, *Precalculus: Enhanced with Graphing Utilities*, p. 24, #51.

Precalculus

Intercepts and Symmetry

Similarly, the graph of $9x^2 + 4y^2 = 36$ is symmetrical about the x -axis if both y and $-y$ give the same x -value. We substitute $-y$ into the equation for y and simplify.

$$9x^2 + 4(-y)^2 = 36$$

and since $(-y)^2 = (-1 \cdot y)^2 = (-1)^2 \cdot y^2 = 1 \cdot y^2 = y^2$ we get

$$9x^2 + 4y^2 = 36$$

Thus the graph of $9x^2 + 4y^2 = 36$ is symmetric with respect to the x -axis.

We can use the information about intercepts and symmetry, along with some other points we can compute, to sketch the graph of the equation $9x^2 + 4y^2 = 36$.

