Sales and Training  The average monthly sales volume (in thousands of dollars) for a company depends on the number of hours of training $x$ of its sales staff, according to

$$S(x) = \frac{20}{x} + 40 + \frac{x}{2} \quad \text{for} \quad 4 \leq x \leq 120$$

a.  Graph this function.

b.  How many hours of training will give average monthly sales of $51,000$?

Average monthly sales are given in thousands of dollars so $51,000$ is represented by the line $y = 51$. The intersection of these two graphs gives us the number of hours of training needed to achieve an average of $51,000$ in monthly sales.

The point (20,51) tells us that 20 hours of training will give average monthly sales of $51,000$.

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1Harshbarger/Yocco, College Algebra In Context, 5e, p. 491, #52.
We can also solve this problem algebraically by letting \( S(x) = 51 \) and solving for \( x \).

\[
S(x) = \frac{20}{x} + 40 + \frac{x}{2}
\]

\[
51 = \frac{20}{x} + 40 + \frac{x}{2}
\]

\[
51(2x) = \frac{20}{x}(2x) + 40(2x) + \frac{x}{2}(2x)
\]

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\]

\[
102x = 40 + 80x + x^2
\]

\[
0 = x^2 - 22x + 40
\]

\[
0 = (x - 2)(x - 20)
\]

\[
x = 2 \quad \text{or} \quad x = 20
\]

But remember, the function is only defined in the interval \( 4 \leq x \leq 120 \) so we must omit the value \( x = 2 \) as a solution.

And again, 20 hours of training will give average monthly sales of $51,000.