Cost The total cost function for a product is given by $C(x) = 3x^3 - 6x^2 - 300x + 1800$, where $x$ is the number of units produced and $C$ is the cost in hundreds of dollars. Use factoring by grouping to find the numbers of units that will give a total cost of $120,000.$

In this problem we need to remember that $C$ is the cost in hundreds of dollars. Since $120,000$ is $1200$-hundred, we want to find the value of $x$ that will result in $C(x) = 1200$.

\[
C(x) = 3x^3 - 6x^2 - 300x + 1800
\]

1200 = 3x^3 - 6x^2 - 300x + 1800 Subtract 1200 from each side so one side is equal to 0.

0 = 3x^3 - 6x^2 - 300x - 600 Divide each term by 3 to simplify the factoring.

0 = x^3 - 2x^2 - 100x - 200 Factor by grouping.

0 = (x^3 - 2x^2) + (-100x - 200)

0 = x^2(x - 2) - 100(x - 2)

0 = (x^2 - 100)(x - 2) Set each factor equal to 0.

\[
x^2 - 100 = 0 \quad \text{or} \quad x - 2 = 0
\]

\[
x^2 = 100 \quad \text{or} \quad x = 2
\]

\[
x = \pm 10
\]

The three algebraic solutions are $x = -10, 2, 10$ but in the context of this problem, where $x$ is the number of units produced, negative values of $x$ must be omitted.

So the solutions are $x = 2, 10$ and we can say that if either 2 units or 10 units are produced the total cost for the product will be $120,000$.

We can verify this result using the graph of $C$. 

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1Harshbarger/Yocco, College Algebra In Context, 5e, p. 465, #42.