Volume of a Cube The volume of a cube is \( f(x) = x^3 \) cubic inches, where \( x \) is the length of the edge of the cube in inches. ¹

a. Is this function one-to-one?

A function \( f \) is one-to-one if each output of the function corresponds to exactly one input in the domain of the function.

A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point. Let’s look at the graph of \( f(x) = x^3 \).

Since any horizontal line will intersect the graph of \( f \) only once, we can say that the function passes the horizontal line test and, yes, it is one-to-one.

b. Find the inverse of this function.

\[
\begin{align*}
    f(x) &= x^3 & \text{Start with the original function, } f(x). \\
    y &= x^3 & \text{Replace } d(x) \text{ with } y. \\
    x &= y^3 & \text{Replace all } x's \text{ with } y's \text{ and all } y's \text{ with } x's. \\
    \sqrt[3]{x} &= \sqrt[3]{y^3} & \text{Solve for } y. \\
    \sqrt[3]{x} &= y \\
    f^{-1}(x) &= x^{\frac{1}{3}} & \text{Replace } y \text{ with the inverse function notation, } f^{-1}(x).
\end{align*}
\]

The inverse of \( f(x) = x^3 \) is \( f^{-1}(x) = x^{\frac{1}{3}} \).

Notice how the graphs of \( f \) and \( f^{-1} \) are reflections of each other across the line \( y = x \).

¹Harshbarger/Yocco, College Algebra In Context, 5e, p. 285, #44.